Center of Mass

Daniel Pinkston

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1 Definition

Average position of mass of an object; all mass of an object can be imageined to be concentrated at that point

2 Finding Center of Mass

Consider meter stick (platonic ideal). Where would center of mass be? Ideally, 50 cm.

Again, consider this amorphous blob:

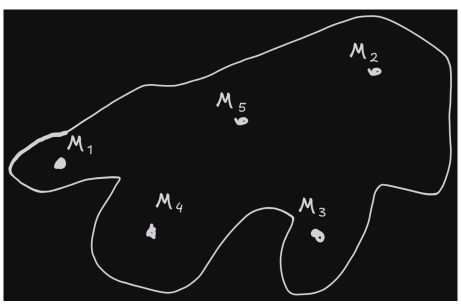

Figure 1: Object is divided into an infinitesimal amount of points. Center of mass is average of each x/y in each coordinate

2.1 Weighted average

$$\frac{m_1x_1 + m_2x_2 + m_3x_3 \ldots + m_ix_i}{m_1 + m_2 + m_3 \ldots m_i} \tag{1}$$

Better written as:

$$\frac{1}{\text{M}}\sum_{i=1}^{N} m_i x_i \tag{2}$$

Variable	Meaning
M	total mass
m_i	each pass point
x_i	each x coordinate
N	number of points (infinite)

3 Center of Mass In Motion

$$\vec{r_{\text{cm}}} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r_i} \tag{3}$$

$\vec{r}$ is position vector of center of mass and its components
What if components can move (ie. waterbottle with 2/3 water)? We shall improve:

$$\Delta\vec{r_{\text{cm}}} = \frac{1}{M}\sum_{i=1}^{N} m_i \Delta\vec{r_i} \tag{4}$$

$\Delta\vec{r}$ is change in position vector
Remember:

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} \tag{5}$$

Extended to:

$$\vec{v_{cm}} = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{\Delta\vec{r}}{\Delta t} \tag{6}$$

Finally:

$$\vec{v_{cm}} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{v_i} \tag{7}$$

Pretty cool aside: $m_i\vec{v_i}$ is momentum (mass $\times$ velocity).

4 Translational Motion

4.1 Definition

rigid body moving from one point in space to another; ex: throwing a ball in a parabola

4.2 Considerations

Anything that is rotating is always accelerating ($\vec{a}$). Must reframe acceleration in our rotating system. Acceleration is 2nd order derivative fo position

4.3 Angular Quantities

Variable	Meaning
θ	Angular position
$r \times \text{d}\theta$	Arc Length/ Angular Displacement
$\frac{\text{d}\theta}{\text{d}t} = \omega$	Angular velocity
$\frac{\text{d}^2\theta}{\text{d}t^2} = \frac{\text{d}\omega}{\text{d}t} = \alpha$	Angular acceleration
$\vec{L} = I \times \vec{\omega}$	Angular momentum
$I = \int r\text{d}m$	Moment of intertia (how hard to move at given angle

4.4 Converting Angular Velocity to Linear Velocity

How can we relate angular/linear velocity of a body in circular motion?

r $\times \theta = s$
$r = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}s}{\mathrm{d}t}$
$r \times \vec{\omega} = \vec{v}$
$r \times \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} r \times \vec{\omega} = \vec{a}$

5 Examples

5.1 Basic

1. 3 2kg masses hug from meter stick. First is 7cm mark second at 12cm, third 25cm. Find center of mass.

$$\frac{2 \times 7 + 2 \times 12 + 2 \times 25}{6} = 14.67 \text{ cm mark} \tag{8}$$

2. From center of mass of sun to center of mass of Earth is 150×10^6 km away. The sun's mass is $1.989 \times 10^3 3$ kg and Earth about $5.97 \times 10^2 4$ kg. Find center of mass of the Earth-sun system.

$$\frac{0 \times (5.97 \times 10^2 4) + (150 \times 10^6)(1.989 \times 10^3 0)}{2} = 1.49175 \times 10^{11} \tag{9}$$

5.2 Translational Motion

1. Record player moves at 0.55 revolutions per second. Convert to radians/seconds.

$$0.55 * 2\pi = 1.1\pi \frac{\mathrm{m}}{\mathrm{s}} \tag{10}$$

2. After stopping, takes 0.5 seconds to stop. Find angular acceleration.

$$\alpha = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1.1\pi}{0.5} = 6.91 \frac{\mathrm{m}}{\mathrm{s}} \tag{11}$$

3. Find angular position after 8 seconds

$$1.1\pi \times 8 = 8.8\pi \text{ radians} \tag{12}$$

4. Tires spins $\frac{1}{5}$ revolutions in 0.05 seconds. Find ω.

$$\omega = \frac{0.4\pi}{0.05} = 8\pi \tag{13}$$

5. Find size of tire if linear velocity of edge of tire is 3 m/s (hint: $\vec{v} \to \vec{\omega}$)

$$s = r \times \theta \tag{14}$$

didn't get the answer for this one, we ran out of time

6 Homework

1. A turntable (radius of 20 cm) is spinning at 2π rad/sec. Find the linear velocities and accelerations of objects placed 5 cm, 11 cm, and 17 cm from the axis of rotation.

   Find velocity using formula: $\vec{v} = r\omega$

   $\vec{v_1} = 0.5 \times 2\pi = 0.1\pi \mathrm{m/s}$
   $\vec{v_2} = 0.11 \times 2\pi = 0.22\pi \mathrm{m/s}$
   $\vec{v_3} = 0.17 \times 2\pi = 0.34\pi \mathrm{m/s}$

   No acceleration because no change in velocity

2. Find the distances traveled by each object in 7 seconds.

   Using formula: $d = \vec{v} \times t$

   $d_1 = 0.1\pi * 7 = 0.7\pi$ meters
   $d_2 = 0.22\pi * 7 = 1.54\pi$ meters
   $d_3 = 0.34\pi * 7 = 2.38\pi$ meters

3. The object 5 cm from the axis is placed $\frac{1}{4}$ π radians from θ=0, the 11 cm object is $\$2_{\overline{3\$x\pi}}$ radians from θ=0, and the 17 cm object is π radians from θ=0. Each object has a mass of 1 kg. Find the center of mass

   First, organize our information: $r_1 = 5$ cm and $\theta_1 = \frac{1}{4}\pi$ $r_2 = 11$ cm and $\theta_2 = \frac{2}{3}\pi$ $r_3 = 17$ cm and $\theta_3 = \pi$

   Use formula:

$$\vec{r_{\text{cm}}} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r_i} \tag{15}$$

   Finding X coordinate:

   $x_1 = 5 * \cos\frac{1}{4}\pi = \frac{5\sqrt{2}}{2}$
   $x_2 = 11 * \cos\frac{2}{3}\pi = 6.5$
   $x_3 = 17 * \cos\pi = -17$

   CoM X coordinate:

$$\vec{r_{\text{cm}}} = \frac{1 \times \frac{5\sqrt{2}}{2} + 1 \times 6.5 + 1 \times -17}{3} = -6.32 \tag{16}$$

   Finding Y coordinate:

   $y_1 = 5 * \sin\left(\frac{1}{4}\pi\right) = 5\frac{\sqrt{2}}{2}$
   $y_2 = 11 * \sin\left(\frac{2}{3}\pi\right) = \frac{11\sqrt{3}}{2}$
   $y_3 = 17 * \sin(\pi) = 0$

   CoM Y coordinate:

$$\vec{r_{\text{cm}}} = \frac{1 \times \frac{5\sqrt{2}}{2} + 1 \times \frac{11\sqrt{3}}{2} + 1 \times 0}{3} = 4.354 \tag{17}$$

   Final Answer:

   Center of mass is $(-6.32, 4.354)$